Number Operations

BODMAS, distributive, associative and commutative laws, equations

1

Lesson Outcomes

US119362 SO 2 AC 3, SO 4 AC 1, SO 5 AC 1-3

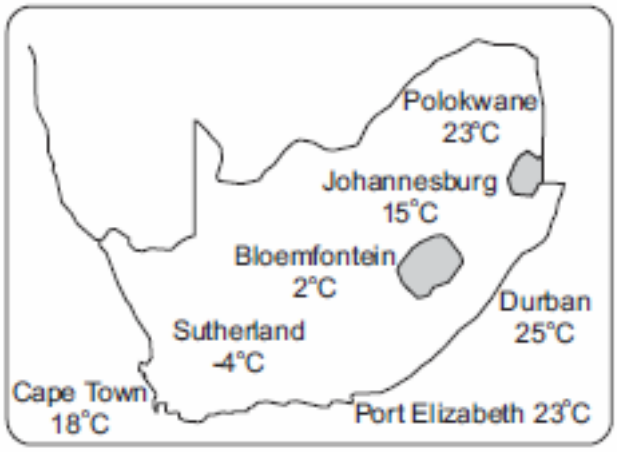
US CCFO Organising: Organising and managing oneself and one's activities responsibly and effectively.

By the end of the lesson, learners should be able to:

- ✓ Revise the four operations in problems with integers.
- ✓ Apply the BODMAS rule.
- ✓ Apply the commutative law.
- ✓ Apply the associative law.
- ✓ Apply the distributive law.

Starting Blocks

This map shows the daytime temperatures in a number of towns in South Africa.



Answer the following questions:

a. Which town is the hottest? Write the temperature.

_____

b. Which town is the coldest? Write the temperature.

_____

c. What is the difference between the temperatures in Polokwane and Port Elizabeth?

_____

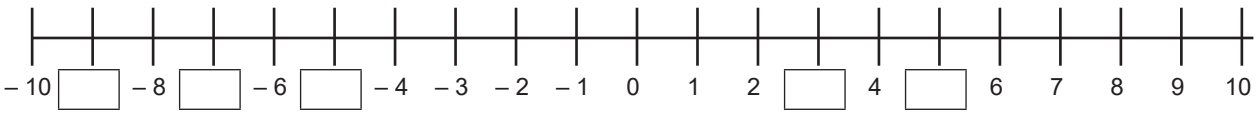
d. What is the difference between the temperatures in Cape Town and Johannesburg?

_____

e. The temperature in Bloemfontein is 2°C during the day. If it drops by 5°C at night, what will the night time temperature be?

_____

Now fill in the missing numbers on the number line.



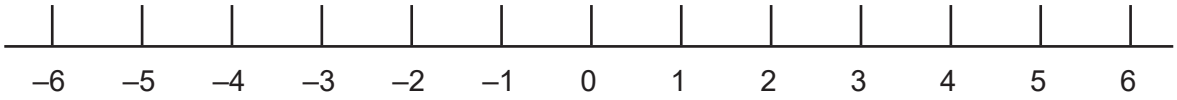
10 marks

Fact File

Integers

Integers are numbers which can be positive or negative or zero. They do not include fractions or decimals. Here are some of the integers: ..., –3, –2, –1, 0, 1, 2, 3, ...

It is useful to show the integers on a number line, especially when adding or subtracting.



Addition of integers

Start from 0 on the number line. If a number is positive, move that many spaces to the right. If a number is negative, move that many spaces to the left.

For example: (–4) + 3: From zero, move 4 spaces to the left, then 3 spaces to the right. The answer is –1.

Subtraction of integers

The original direction is changed.

For example: When adding (2) + (–5), the direction is 2 to the right, followed by 5 to the left.

However if these integers are subtracted, the direction must change:

(2) – (–5) becomes 2 to the right, followed by 5 to the right.

Note that the presence of a negative sign in front of a bracket will change the sign of the number in the bracket. Thus (2) – (–5) = 2 + 5

Individual Tasks

Exercise 1

Give the answers to the following addition and subtraction questions.

a.	3 + (– 9)	=	
b.	5 – (– 8)	=	
c.	20 + (– 8)	=	
d.	– 30 + (– 8)	=	
e.	– 7 – (– 8)	=	
f.	35 – 82	=	

6 marks

Exercise 2

In these questions, first write an expression to calculate the answers. Remember that in a word problem, you must always give a number sentence first, then give the final answer in words.

a. If you are at –100 m in a mine shaft, and the lift takes you down another 130 m, write an expression and calculate your new level.

(2)

b. Your bank account is at –R1 200 and you deposit R500. A week later you withdraw R200. Write an expression and calculate your new bank balance.

(2)

c. The midnight temperature in Vereeniging is –8°C and that in Nelspruit is 12°C. Find out how much higher Nelspruit's temperature is.

(2)

6 marks

Fact File

Multiplication of integers

Example: Suppose Dikeledi borrows R25 every day from Monday to Friday, what is her total debt? This could be written as (–25) + (–25) + (–25) + (–25) + (–25). The answer to this repeated addition is (–125). Dikeledi has a total debt of R125. This can also be done as multiplication:

5 x (–25) = –R125

Fact File

Division of integers

Example: Suppose Anna kept borrowing R25 every day for some time, and eventually her total debt came to R375. How many days did she borrow money? On the number line, start at 0 and move 25 spaces to the left. Repeat this until you reach – 375. The answer is fifteen times. This can also be done as division: (–375) ÷ (–25) = 15. The answer is 15 days.

The rules for signs for multiplication and division are the same. If we multiply or divide two negatives, it gives us a positive. If we multiply or divide one positive and one negative, the answer is negative.

Exercise 3

Give the answers to the following multiplication and division questions.

a.	(– 32) ÷ 8	=	
b.	(– 4) x (– 3)	=	
c.	(– 36) ÷ (– 2)	=	
d.	7 x (– 9)	=	

(4)

e. Alu's bank account contains R1 300. He makes 6 withdrawals of R230 each.

(i) What is Alu's new bank balance?

(1)

(ii) Alu can only pay back R30 per month. After how many months will he be able to clear his debt?

(1)

6 marks

Fact File

Mathematical laws: BODMAS

The order in which we do calculations is very important.

First do any **B**rackets, then '**O**f' or '**O**rder', **D**ivide or **M**ultiply (from left to right), then finally **A**dd or **S**ubtract (from left to right).

The commutative law

This law is about whether we can swop the numbers around or not when we are doing an operation.

- Addition is commutative: 6 + 3 and 3 + 6 both give the same answer, 9.
- Subtraction is not commutative: 6 – 3 = 3 but 3 – 6 = – 3. The answers are not the same.
- Multiplication is commutative: 2 x 3 and 3 x 2 both give the same answer, 6.
- Division is not commutative: 6 ÷ 3 = 2 but 3 ÷ 6 = 0,5. The answers are not the same.

The associative law

This law is about how we group numbers together using brackets. Once again, only addition and multiplication have this law.

- For addition: (3 + 5) + 2 = 8 + 2 and 3 + (5 + 2) = 3 + 7. Both give an answer of 10.
- For subtraction: (8 – 5) – 2 = 3 – 2 which is 1, but 8 – (5 – 2) = 8 – 3 which is 5. The answers are not the same.
- For multiplication: (3 x 5) x 2 = 15 x 2 and 3 x (5 x 2) = 3 x 10. Both give an answer of 30.
- For division: (20 ÷ 10) ÷ 2 = 2 ÷ 2 which is 1, but 20 ÷ (10 ÷ 2) = 20 ÷ 5 which is 4.

The answers are not the same.

The distributive law

This is a very useful law, and it combines addition (or subtraction) and multiplication.

The distributive law says that we can 'distribute' the number in front of a bracket by multiplying it separately by the terms inside the bracket.

e.g. 3(4 + 2) = 3(4) + 3(2)

= 12 + 6 = 18

Exercise 4

In the exercise below, each of the answers is wrong. Give the correct answer and say which law or rule should have been applied. Select the correct rule from the box below.
(Give the correct letter.)

A. The product of two negatives is a positive.

B. When dividing, opposite signs give a negative answer.

C. A negative sign in front of a bracket will change the sign of the number in the bracket.

D. For the distributive law, multiply the number in front of the bracket by both numbers in the bracket.

E. BODMAS

F. The associative law does not work for subtraction.

	Wrong answer	Correct answer	Rule
a.	5(7 + 2) = 35 + 2 = 37		
b.	(–5) x (–3) = –15		
c.	13 – 3 x 2 = 10 x 2		
d.	(8 – 1) – 3 = 8 – (1 – 3)		
e.	(–250) ÷ 2 = 125		
f.	3 – (–8) = –5		

(12 x ½)

6 marks

Exercise 5

Ndumiso leaves home and travels 8 km in his taxi. He stops at the taxi rank to pick up passengers then travels to town and back to the taxi rank three times. At this point he has travelled a total of 86 km. Find how far it is from the taxi rank to town.

3 marks

Fact File

Binary numbers

Computer circuits use the numbers 0 or 1 to show whether a circuit is on or off. The on/off switch on many electronic devices has a 1 and 0 on top of each other like this: 

The binary number system was developed for use in computers. Instead of using 10 digits like the decimal system, it uses only two digits: 1 and 0. The table shows how binary numbers are written. The word 'binary' means two, and binary numbers often have a small 2 written next to them. Look at the pattern in the table.

1	2	3	4	5	6	7	8	9
1	10_2	11_2	100_2	101_2	110_2	111_2	1000_2	1001_2

Exercise 6

1. Complete the pattern with binary numbers.

   2 x 1 = 2 x 2 = 2 x 2 x 2 = 2 x 2 x 2 x 2 =

   (4)

2. Can you work out the number that is written as **10001_2**? (Hint: it is one more than the last number you worked out in question 1).

   (1)

5 marks

$\overline{42}$ = %

Glossary

associative law	numbers can be grouped in different ways for addition and multiplication, e.g. (2 + 3) + 4 = 2 + (3 + 4) and (2 x 3) x 4 = 2 x (3 x 4)
bank balance	the amount of money left in the bank
BODMAS	an acronym giving the order in which calculations must be done. Brackets, 'of', divide or multiply, then add or subtract, from left to right
commutative law	order doesn't matter for addition and multiplication e.g. 3 + 4 = 4 + 3 and 3 x 4 = 4 x 3
debt	money that is owing
distributive law	the number outside a bracket must be multiplied by both terms inside the brackets
brackets	a pair of marks used to enclose numbers or words e.g. 5(6 + 7) = 5 x 6 + 5 x 7
expression	a statement in numbers and operations, e.g. 6 + 3 – 2
integers	...–3, –2, –1, 0, 1, 2, 3, ...
negative	a number which has a minus sign in front of it; the opposite of positive
operations	e.g. +, –, x, ÷
positive	a number with a plus sign (or no sign) in front of it; the opposite of negative
product	the answer when multiplying
terms	numbers separated by + or – signs. e.g. the expression 4 + 2 – 15 has three terms
unlike signs	opposite signs, one minus and one plus
withdraw	to take money out of a bank account

Facilitator Feedback

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Extra Tasks

Task 1

Give the answers to the following addition and subtraction questions.

a. 17 + (– 9) = _____

b. 3 + (– 9) = _____

c. 3 – (– 9) = _____

d. – 3 – (– 9) = _____

e. –11 + (– 9) = _____

5 marks

Task 2

a. If you are –300 m underground in a mine shaft, and the lift takes you down another 150 m, what will your new level be?

_____

_____

(2)

b. If the midnight temperature in Johannesburg is –5°C and in Durban it is 13°C. How much higher is Durban's temperature?

_____

_____

(2)

c. You owe Telkom R350 on your telephone account. You pay Telkom R150, but then you make telephone calls amounting to R175 over the next month. How much will you owe Telkom next month?

_____

_____

(2)

6 marks

Task 3

Give the answers to the following multiplication and division questions.

a. 60 ÷ (–12) = _____

b. (–40) x (– 8) = _____

c. (–56) ÷ (–7) = _____

d. (–8) x 8 = _____

(4)

e. Simon's bank account contains R2 500. He makes 9 withdrawals of R300 each.

(i) What is Simon's new bank balance?

_____

_____

_____

_____

(3)

Task 4

Do the calculations and give the correct answers. Show all your steps.

a. 3(7 – 3 + 1)

b. 10 – 3 x 8

c. (7 – 1) – 2

3 marks

Task 5

Thulani had R200. He spent a total of R24 on transport and bought five shirts. When he got home he had R26 left. Find out how much Thulani paid for one shirt.

4 marks

$\overline{25}$ = %

Notes

